Statistical Analysis Using Combined Data Sources: Discussion
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Michael Elliott

1University of Michigan School of Public Health

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“Complete” (Ideal) vs. Observed (Messy) Data

- “Complete” data $d_U$ vs. observed data $d_s$:
  - Data with measurement error ($X$ vs. $X^*$): last month vs. this month employee count.
  - Partial $Z$: “large” area IDs vs. small area IDs.
  - Non-overlapping covariates: health outcomes $X$ vs. behavioral risk factors $W$.
  - Multiple surveys.
Would like to make inference about 
\[ Q(Y_1, \ldots, Y_N, X_1, \ldots, X_N) = Q(X, Y) \] using \( d_U \)

Stuck with making inference about \( Q(X, Y) \) using \( d_s \)

Chambers: Use “missing information principle” to obtain likelihood inference about \( \theta \equiv \theta_N \equiv Q(X, Y) \) using \( d_s \) by replacing sufficient statistics \( S(d_U) \) for \( \theta_N \) by \( E(S(d_U) | d_s) \).

EM algorithm

Replace likelihood inference with pseudo-likelihood inferences to accommodate unequal probability sample designs.
Missing Information Principle

- Alternative to calibration: calibration data becomes part of $d_s$, and replace data in MLE/PMLE score equations with expected value conditional on calibration totals.
  - Chambers shows that this can improve efficiency over uncalibrated estimators, and avoid bias due to failure of the implied calibration model.
- Probabilistic linkage: allows for bias correction in estimating equations using probabilistic linked datasets.
- Combining data from multiple surveys using MIP extension of missing data algorithms.
Bayesian Survey Inference

Focus on inference about $Q(X, Y)$ using posterior predictive distribution based on $p(Y_{nobs}, X_{nobs} \mid y, x)$:

$$p(Y_{nobs}, X_{nobs} \mid y, x) = \frac{p(Y, X)}{p(y, x)} =$$

$$= \frac{\int p(Y, X \mid \theta)p(\theta)d\theta}{p(y, x)}$$

$$= \frac{\int p(Y_{nobs}, X_{nobs} \mid y, x, \theta)p(y, x \mid \theta)p(\theta)d\theta}{p(y, x)}$$

$$= \int p(Y_{nobs}, X_{nobs} \mid y, x, \theta)p(\theta \mid y, x)d\theta$$

(Ericson 1969; Scott 1977; Rubin 1987).
Both Bayesian survey inference and the missing information principle focus on prediction of missing elements in the population conditional on observed data.

Bayesian approach obtains full posterior predictive distribution of $Q(X, Y)$, rather than point estimate and asymptotic normality assumptions

- EM vs. MCMC (“stochastic EM”)

Bayesian survey inference should yield similar inference to MIP, at least in large samples.
Applications of Bayesian Survey Inference

- (Item level) missing data (Rubin 1987, Little and Rubin 2002).
- Disclosure risk (Raghunathan et al. 2003, Reither 2005).
- Combining data from multiple surveys (Raghunathan et al. 2007, Davis et al. 2010).
Motivating example: obtain inference about $Q(Y, X, W)$ when only two of the three variables are contained in any one survey.

Surveys use different designs and data collection methods → different sampling and nonsampling error properties

Cannot simply pool data for analysis.
Combining Data from Multiple Complex Sample Design Surveys (Dong 2011)

- Borrow from disclosure risk literature to generate synthetic populations using data from each survey.
- Each generated population “uncomplexes” sample design to create what is effectively a simple random sample.
Pool data and use standard imputation approaches to fill in missing variables for the data from each survey.
Suppose we have synthetic populations $P_l$, $l = 1, \ldots, L$ generated from $P(Y_{nobs}, X_{nobs} | y, x)$.

Estimate $Q = Q(X, Y)$ with $\bar{Q}_L = L^{-1} \sum_l Q_l$, where $Q_l$ is an (asymptotically) unbiased estimator of $Q$ generated from $P_l$.

$\text{var}(\bar{Q}_L) = T_L = (1 + L^{-1})B_L - \bar{U}_L$, where

$B_L = (L - 1)^{-1} \sum_l (Q_l - \bar{Q}_L)(Q_l - \bar{Q}_L)^T$ is between-imputation variance and $\bar{U}_L = L^{-1} \sum_l U_l$ is the average of the within-imputation variances $U_l$ of $Q_l$.

$T_L \approx B_L$ if generated sample large enough that $U_L$ can be ignored and number of synthetics $L$ large enough that $1/L$ can be ignored.

Need a bit more care if synthetic populations are generated from different surveys, since posterior predictive distribution models from different survey may not be the same.
Generate synthetic populations $\mathcal{P}_i^s$, $l = 1, \ldots, L$ for surveys $s = 1, \ldots, S$ from $P(\mathbf{Y}_{nobs}^s, \mathbf{X}_{nobs}^s \mid y^s, x^s)$.

- Account for complex sample design features; regard synthetic data as SRS from population.
- No need to actually generate entire population, just sample large enough that between-imputation variance swamps within-imputation variance.
For each survey, obtain $\overline{Q}^s_L$ as the synthetic population point estimator of $Q$ for survey $s$ and $B^s_L$ as its variance. If

$\hat{Q}^s \sim N(Q, U^s)$

$Q^s \mid y^s, x^s \sim N(\hat{Q}^s, B_s)$, then

- As $L \to \infty$, posterior predictive distribution of $Q$ approximately $N(\overline{Q}_\infty, B_\infty)$, $\overline{Q}_\infty = \frac{\sum_s \overline{Q}^s / B^s_s}{\sum_s 1/B^s_s}$, $B_\infty = \frac{1}{\sum_s 1/B^s_s}$.

- $t$ approximation available for small $L$. 
Combining Estimates from Synthetic Populations Generated by Multiple Surveys: Extension to Missing Data (Dong et al. 2011)

- Synthesize, then impute.
- Imputation for missing components in each survey obtained by stacking $P_i^s$, $s = 1, \ldots, S$ and treating as SRS from population.
- Multiply impute $m = 1, \ldots, M$ complete datasets for each of the $L$ synthetic populations.
- Resepare into the surveys and obtain $Q_{ml}$ for each of the multiply imputed synthetic datasets.
  - $Q_L^s = L^{-1} \sum_i Q_{M,i}^s$ for $Q_{M,i}^s = M^{-1} \sum_m Q_{ml}^s$.
  - $B_L^s = \frac{1+L^{-1}}{L-1} \sum_i (Q_{M,i}^s - Q_L^s)^2 + \frac{1+M^{-1}}{L} \sum_i (M-1)^{-1} \sum_m (Q_{ml}^s - Q_{M,i}^s)^2$
- Combine survey-level predictive distributions as in the complete data case on previous slide.
Generating Synthetic Populations from Posterior Predictive Distribution

Derivation of predictive distribution ignores sampling indicator $I$; this requires (Rubin 1987)

- Unconfounded sampling
  - $P(I \mid Y, X) = P(I \mid y, x)$
- Independence of $I$ and $(Y_{nobs}, X_{nobs})$ given $y, x, \text{ and } \theta$
  - $P(Y_{nobs}, X_{nobs} \mid y, x, I, \theta) = P(Y_{nobs}, X_{nobs} \mid y, x, \theta)$

Maintaining these assumption requires:

- Probability sample
- Model $p(Y, X)$ attentive to design features and robust enough to sufficiently capture all aspects of the distribution of $Y, X$ relevant to $Q(Y, X)$. 
Generate the \( l \)th synthetic population as follows:

- Account for stratification and clustering by drawing a Bayesian bootstrap sample of the clusters within each stratum.
  - For stratum \( h \) with \( C_h \) clusters, draw \( C_h - 1 \) random variables from \( U(0, 1) \) and order \( a_1, \ldots, a_{C_h-1} \); sample \( C_h \) clusters with replacement with probability \( a_c - a_{c-1} \), where \( a_0 = 0 \) and \( a_{C_h} = 1 \).

- Use finite population Bayesian bootstrap Polya urn scheme (Lo 1988) extended to account for selection weights (Cohen 1997) to generate unobserved elements of the population within each cluster \( c \) in stratum \( h \):
  - Draw a sample of size \( N_{ch} - n_{ch} \) by drawing \((y_k, x_k)\) from the \( i \)th unit among the \( n_{ch} \) sampled elements with probability \( \frac{w_i - 1 + l_{i,k-1} \ast (N_{ch} - n_{ch})}{N_{ch} - n_{ch} + (k-1) \ast (N_{ch} - n_{ch})} \) where \( l_{i,k-1} \) is the number of bootstrap draws of the \( i \)th unit among the previous \( k - 1 \) bootstrap selections.
  - Repeat \( F \) times for each boostrapped cluster.
Application: Distribution of Insurance Coverage

- Estimating 2006 insurance coverage using Behavior Risk Factor Surveillance Survey (BRFSS), National Health Interview Survey (NHIS), Medical Expenditure Panel Survey (MEPS).
- NHIS and MEPS split insured into private and public: impute BRFSS using gender, race, region, education, age, and income.
- Generate synthetic populations using non-parametric Bayesian bootstrap method.
### Application: Distribution of Insurance Coverage

<table>
<thead>
<tr>
<th>Type of coverage</th>
<th>NHIS (n=76K)</th>
<th>BRFSS (n=356K)</th>
<th>MEPS (n=34K)</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point Est. (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Private</td>
<td>74.6</td>
<td>73.4</td>
<td>75.9</td>
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<tr>
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<td>7.5</td>
<td>13.3</td>
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<tr>
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<tr>
<td>None</td>
<td>.43</td>
<td>.18</td>
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</tbody>
</table>
The Role of the Model

- Bayesian bootstrap is very robust: in most settings it doesn’t provide efficiency gains over design-based methods, but “combining data” situations are likely exceptions.

- In application, some issues arise:
  - BRFSS estimator may be biased (low response rate, telephone-only sampling frame).
  - MEPS is a subsample of previous year’s NHIS.
  - More traditional modeling approaches needed?

- MIP uses $E \{ \partial_\theta \log f(d_u) \mid d_s \}$
  - Target quantity of interest if model is misspecified? Solution to population score equation? Can we think of $\theta \equiv \theta_N \equiv Q(X, Y)$?
  - Use estimating equation methodology to obtain consistent estimators of $\theta$. 

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